Name: _____

Student ID: _____

Section: _____

Instructor: _____

Math 113 (Calculus 2)
Exam 3
October 30 – November 3, 2009

Instructions:

1. Work on scratch paper will not be graded.
2. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
3. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, $\tan^{-1}(1)$, etc. must be simplified for full credit.
4. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
M.C.	40			14	16	
11	9			15	6	
12	9			16	6	
13	9			17	5	
				Total	100	

Multiple Choice (40 points). Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. Find the length of the curve $y = \ln(\cos x), 0 \le x \le \pi/4$.

   A. $\ln(1+\sqrt{2})$ B. $\ln(1+\sqrt{3})$ C. $\ln\sqrt{3}$ D. $\ln(2+\sqrt{2})$ E. $\ln(2+\sqrt{3})$ F. None of these.

2. Find the length of the curve $y = \dfrac{x^2}{4} - \dfrac{1}{2}\ln x, 1 \le x \le 2$.

   A. $1+\ln 2$ B. $\dfrac{3}{4}+\dfrac{1}{2}\ln 2$ C. $1+\dfrac{1}{2}\ln 2$ D. $\dfrac{3}{4}+\ln 2$ E. $\dfrac{5}{4}+\dfrac{1}{2}\ln 2$ F. None of these.

3. Find the surface area if the line segment from (3,3) to (7,0) in the x-y plane is rotated about the y-axis. Hint: First compute the length of the line segment.

   A. 30π B. 40π C. 50π D. $50\sqrt{5}\pi$ E. $50\sqrt{7}\pi$ F. None of these.

4. What is the surface area when the arc given by $y = \sqrt{4-x^2}$ for $1 \le x \le 2$ is rotated about the x-axis?

   A. 2π B. 4π C. $4\pi\sqrt{3}$ D. $4\pi\sqrt{5}$ E. 8π F. None of these.

5. What is the surface area when the arc given by $y = \sqrt{4-x^2}$ for $1 \le x \le 2$ is rotated about the y-axis?

   A. 2π B. 4π C. $4\pi\sqrt{3}$ D. $4\pi\sqrt{5}$ E. 8π F. None of these.

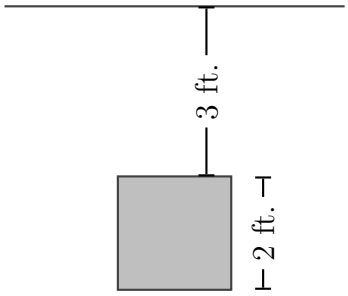
6. Write an integral that represents the surface area when the curve $y = \tan x$, $0 \le x \le \pi/4$ is revolved about the line $x = -2$.

   A. $2\pi\displaystyle\int_0^{\pi/4} (\tan x)\sqrt{1+\sec^4 x}\,dx$ B. $2\pi\displaystyle\int_0^{\pi/4} (\tan x+2)\sqrt{1+\sec^4 x}\,dx$

   C. $2\pi\displaystyle\int_0^{\pi/4} x\sqrt{1+\sec^4 x}\,dx$ D. $2\pi\displaystyle\int_0^{\pi/4} (x+2)\sqrt{1+\sec^4 x}\,dx$

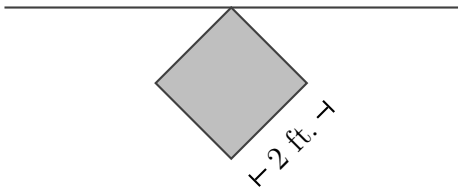
   E. $2\pi\displaystyle\int_0^{\pi/4} (\tan x-2)\sqrt{1+\sec^4 x}\,dx$ F. None of these.

7. What is the hydrostatic force on a 2 foot by 2 foot square aquarium window whose top is 3 feet below the surface of the water if the density of water is 62.5 lbs/ft^3?



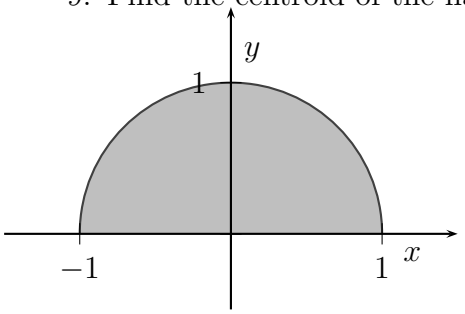
A. 250 lbs. B. 300 lbs.
C. 400 lbs. D. 500 lbs.
E. 750 lbs. F. 1000 lbs.

8. What is the hydrostatic force on a 2 foot by 2 foot square diamond aquarium window whose top is at the surface of the water if the density of water is 62.5 lbs/ft^3?



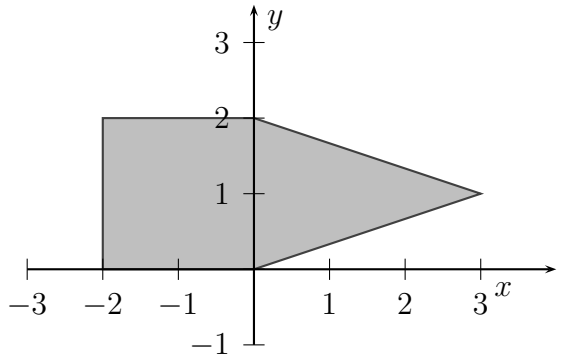
A. 125 lbs. B. $125\sqrt{2}$ lbs.
C. 250 lbs. D. $250\sqrt{2}$ lbs.
E. 500 lbs. F. $500\sqrt{2}$ lbs.

9. Find the centroid of the half-disk of radius 1 in the x-y plane.



A. $(0, \frac{4}{3\pi})$ B. $(0, \frac{2}{3\pi})$
C. $(0, \frac{\pi}{10})$ D. $(0, \frac{1}{3})$
E. $(0, \frac{2}{5})$ F. None of these.

10. Find the centroid of the following system consisting of a square and an isosceles triangle.



A. $(-\frac{1}{5}, 1)$ B. $(-\frac{1}{6}, 1)$
C. $(-\frac{1}{7}, 1)$ D. $(0, 1)$
E. $(\frac{1}{7}, 1)$ F. None of these.

Short Answer (27 points). Fill in the blank with the appropriate answer. 3 points each. A correct answer gets full credit. You will need to show your work for partial credit.

11. (a) Give the definition of $\lim_{n\to\infty} a_n = L$

_____

(b) A sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = 4 - \dfrac{1}{a_n}$ for $n \geq 1$. Assuming that the sequence is convergent, find its limit.

_____

(c) Find the tenth partial sum S_{10} for the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}$.

_____

12. Evaluate the following limits if they exist. If the limit does not exist, so state.

(a) $\displaystyle\lim_{n\to\infty} \frac{n!}{2^n} =$ _____

(b) $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n =$ _____

(c) $\displaystyle\lim_{n\to\infty} \frac{n^2+2}{n^3} =$ _____

13. Determine whether each series converges or diverges. If it converges, give its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} =$ _____

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} =$ _____

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}} =$ _____

Free Response (33 points). Show all of your work and write the final answer in the blank.

14. Determine whether each series converges or diverges. State any convergence/divergence tests you use. For the Integral Test, evaluate the appropriate integral. For the Comparison Test or Limit Comparison Test give the appropriate comparison series.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+1}{n^3+2n^2+n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2n^2+n+1}}$

15. How many terms in the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ are needed to find the sum to within $\dfrac{1}{1000}$?

Answer: _____

16. A region with area 4 lies in the first quadrant of the x-y plane. When the region is revolved about the x-axis, it sweeps out a volume of 12π. When revolved about the y-axis, it sweeps out a volume of 8π. Find the centroid of the region.

Answer: _____

17. Give an example of a series where the terms go to zero, but the series diverges.

Answer: _____